THE General Radio EXPERIMENTER

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ELECTRICAL MEASUREMENTS AND THEIR INDUSTRIAL APPLICATIONS

Also IN THIS ISSUE

IMPEDANCE BRIDGES ASSEMBLED FROM LABORATORY PARTS

● **ON MANY OCCASIONS** when the need for accurate impedance measurements arises, a commercial self-contained bridge is not available, and its purchase may not be economically justified. It is generally possible, however, using standard components that are readily available in the average laboratory, to set up bridge circuits on the bench that, at audio frequencies, will realize accuracies approaching those of commercial bridges over a wide range of impedances. It must be recognized, however, that some sacrifice of convenience and speed of measurement will necessarily be involved.

The unavoidable stray capacitances that are associated with such circuits lead many laboratory workers to view them with considerable suspicion, particularly for the measurement of high impedances or of small quadrature components. But, with shielded components, and with

(*Continued on page 2*)

PRIORITIES

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a good bridge transformer, such as the Type 578 Shielded Transformer, the disposition of the stray capacitances can be definitely controlled. The magnitudes of these capacitances can then be determined with considerable accuracy and allowance made for their effects so that the accuracy of measurement will depend mainly upon the accuracy of the standards and of the bridge arms.

On the following pages are described bench setups of a number of conventional bridge circuits using standard components, and the disposition and measurement of the various stray and residual circuit impedances are discussed.

CAPACITANCE BRIDGES

Figure 1 shows a generalized capacitance bridge circuit and the complete equations of balance. This circuit can represent any one of a number of well-known capacitance bridge circuits, depending upon the relative magnitudes of the various impedances shown. For example, if capacitances C_A and C_B are zero, we have the familiar series-resistance type of capacitance bridge, wherein the losses in the P arm are balanced by the resistance N in the standard arm. This circuit is used in the General Radio Type 740 Capacitance Test Bridge, with the resistance R_N calibrated directly in dissipation factor* at 60 cycles. It is also used in the Type 650-A Impedance Bridge. Here again the N arm resistance is calibrated to read directly in dissipation factor, in this case at 1000 cycles.

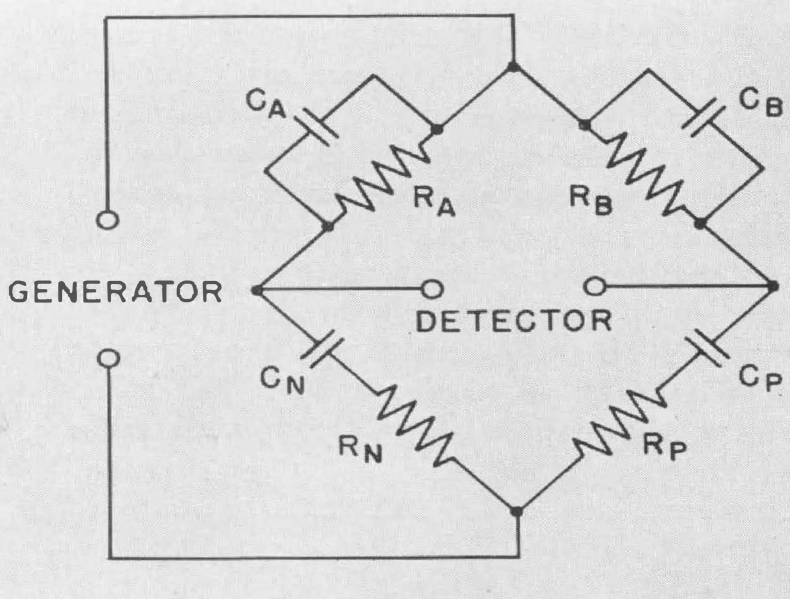
As another example, if C_N and C_P are made infinite, the network reduces to the parallel-resistance type capacitance bridge in which the standard and unknown arms are A and B. In this circuit the effective parallel resistance of the unknown capacitance is balanced by the resistance, R_A, in parallel with the standard capacitor, C_A.

If D_N and Q_B are small compared to D_P and Q_A the network assumes the form of a Schering bridge, which is characterized by the fact that the dissipation factor of the P arm is balanced by parallel capacitance in the opposite arm. This is the circuit used in the Type 716

*The ratio of series resistance to series reactance, equal to $R\omega C$ for a condenser in series with a resistance. The symbol D, with appropriate subscripts, will be used throughout for this quantity.

The ratio of series reactance to series resistance, frequently called storage factor, will be designated as Q. For a capacitance in parallel with a resistance, we have $Q = R\omega C$.

Figure 1. Generalized capacitance bridge circuit, with equations of balance. Note that the expressions are written in terms of the dissipation and storage factors of the arms, so that frequency does not enter explicitly.



$$Q_A = R_A \omega C_A \qquad D_P = R_P \omega C_P$$

$$Q_B = R_B \omega C_B \qquad D_N = R_N \omega C_N$$

$$C_P = C_N \left(\frac{A}{B}\right) \frac{1 + Q_B^2}{1 + D_N(Q_B - Q_A) + Q_A Q_B}$$

$$D_P = \frac{D_N + Q_A - Q_B(1 - Q_A D_N)}{1 + D_N(Q_B - Q_A) + Q_A Q_B}$$

Capacitance Bridge, a highly precise direct-reading bridge for the measurement of capacitance and dissipation factor.

The fundamental equations of balance shown hold for all the reduced circuits mentioned, and, if accurate results are to be obtained, the complete expressions must be retained and examined for the effect of stray capacitances and other residual impedances in the bridge arms.

GENERAL CONSIDERATIONS

In the circuit shown in Figure 1 the generator and detector terminals are shown merely as two pairs of terminals brought out from opposite corners of the bridge. So far as the bridge balance equations are concerned, it is immaterial whether generator and detector are connected as shown or are interchanged. Their location is usually governed by considerations of sensitivity, the connections being made in the manner which yields the maximum output voltage for a given input voltage and a given unbalance of the bridge.

Inasmuch as high-input-impedance amplifiers preceding the null detector are almost universally used in a-c bridge measurements, the sensitivity problem is best discussed on the basis of the use of a detector of infinite impedance. It can be shown[1] that for a given unbalance of the bridge, the ratio of open-circuit output voltage to input voltage (with the generator across a pair of resistive arms) is

(1) $$\frac{E_o}{E_i} = \frac{\frac{A}{B}}{\left(1 + \frac{A}{B}\right)^2} d$$

where A and B are the resistance of the arms across which the generator is connected, and d is the fractional change in the unknown from the condition of true balance. If the generator is connected across unlike bridge arms (one resistive, the other reactive) Equation (1) becomes

(2) $$\frac{E_o}{E_i} = \frac{\frac{A}{B}}{1 + \left(\frac{A}{B}\right)^2} d$$

where either A or B is a reactance.

Either the detector may be grounded and the generator connected to the bridge through a shielded transformer or vice versa. When the detector is grounded, its capacitance to ground becomes part of its own terminal impedance, and does not affect the bridge balance. The terminal-to-ground capacitances of the generator are replaced by those of the shielded transformer. Although these capacitances appear across the bridge arms,* they are small, localized, and measurable; and allowance can be made for their effects on the measurement. When the generator is grounded, the transformer is used between the detector and the bridge.

Two of the points at which the bridge may be grounded leave both sides of the unknown above ground potential, while grounding either of the other two points grounds one side of the unknown capacitance. The choice of the point of grounding will depend somewhat upon the type of measurement that it is desired to make. For capacitors that are physically large, and hence subject to electrostatic pickup and stray capacitance effects, or for unknowns that have one side normally grounded, it is preferable to ground the bridge at one of the unknown terminals. On the other hand, if both unknown terminals are above ground it is possible to measure direct

*By the use of Wagner grounds or guard circuits the effect of these impedances may be removed from the measuring circuit. This subject will be treated in some detail in a later issue.

capacitance between any two terminals of a three-terminal capacitance.

Extraneous coupling from the voltage source* to the bridge arms, to the unknown, or to the ungrounded detector terminals can cause serious errors in direct-reading measurements, and second-order errors in substitution measurements. Difficulties from this cause can be largely overcome by using well shielded oscillators and amplifiers and connecting them to the bridge with shielded leads.

In order to obtain maximum sensitivity and ease of balance it is desirable to use a tuned detector to eliminate harmonics, hum, and residual noise, which may otherwise obscure the fundamental balance and make precise balances difficult or impossible.

THE SERIES-RESISTANCE BRIDGE

Perhaps the simplest and most straightforward of the bridges mentioned is the series-resistance bridge, using variable ratio arms to balance the unknown capacitance against a fixed standard and a variable resistance in series with the standard condenser to balance the losses in the unknown arm. The arrangement illustrated in Figure 2, using Type 602 Decade-Resistance Boxes, a Type 578-A Transformer, and a Type 509 Standard Condenser has been found very satisfactory for measurements over a wide range of capacitance and power factor.

With this bridge, accurate results can be obtained if the various circuit and circuit-element residuals are measured and their effects on the capacitive and resistive balances computed.

CAPACITIVE BALANCE

Circuit Residuals

The capacitance C_{sg}, consisting of inter-shield capacitances of the transformer winding and the capacitance or point (1) to ground, appears across the N arm and causes an error depending directly on the ratio of its magnitude to that of the standard. Across the unknown arm P is placed C_{Po}, which is the capacitance of point (2) to ground.

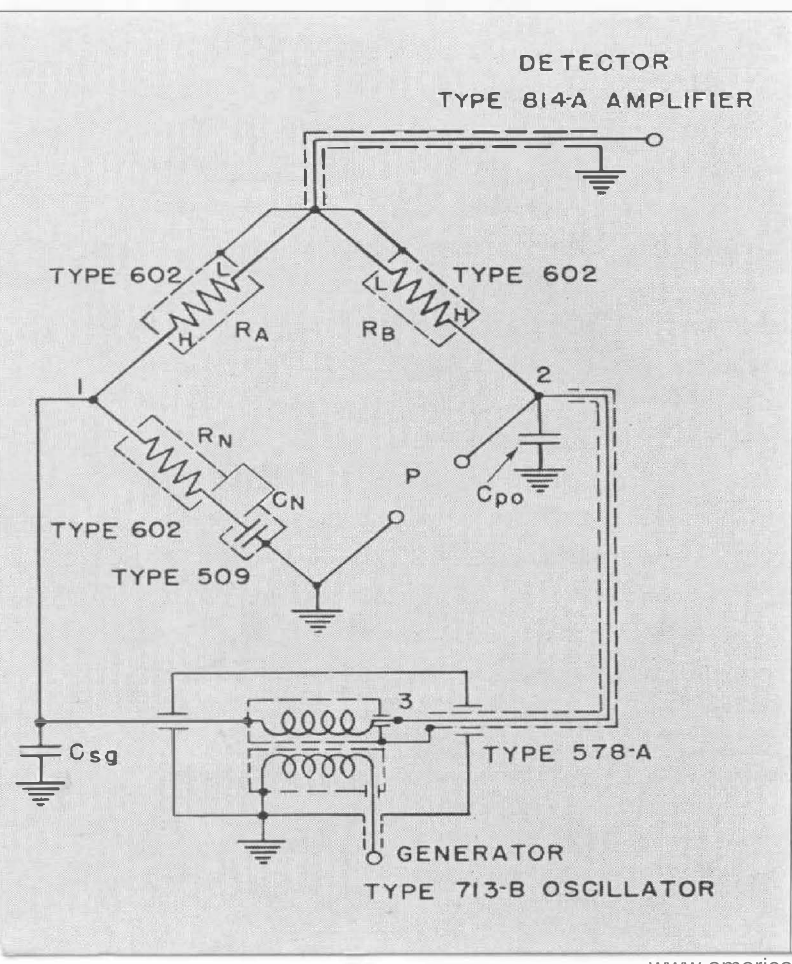
The magnitudes of the two capacitances thus placed across the lower arms of the bridge can be determined quite accurately by balancing the bridge with C_{sg} connected alternately across the N and P arms. With the connections shown, C_{Po} is measured directly (if the standard

*The effect of the extraneous voltage introduced into the bridge circuit by a capacitance or resistance coupling between the windings of the transformer is particularly interesting and will be discussed in some detail in a later issue.

Figure 2. Connections for a bench layout of a series-resistance type of capacitance bridge. The approximate equations of balance are

$$C_P = C_N \frac{A}{B}$$

$$D_P = D_N + Q_A - Q_B$$



condenser is large compared to C_{sg}); with the leads from the transformer reversed at the bridge, a value for C_{sg} is obtained that will be in error by the amount of capacitance contributed by the point (1) to ground. With careful wiring and arrangement of components, this capacitance will be small. The error in the measurement of these quantities probably does not exceed a micromicrofarad for C_{Po} and five micromicrofarads for C_{sg}. Typical values are 100 $\mu\mu$f for C_{sg} and 10 $\mu\mu$f for C_{Po}. In general, it is probably more desirable to use the circuit with C_{sg} across the N arm. If the capacitance of the standard is 0.01 μf, the resulting correction for C_{sg} will be of the order of 1%, subject to an error of only a few hundredths per cent.

INTERNAL RESIDUALS IN STANDARDS

Next in order of importance are the terminal-to-shield capacitances of the decade-resistance boxes. The TYPE 602 Decade-Resistance Boxes are completely shielded, with a separate shield terminal provided on the panel. Figure 3 represents, to a first approximation, the shielded decade box and the associated capacitances. The terminal connected to the highest resistance decade is designated as H, while the terminal connected to the lowest resistance decade, and located nearest to the shield terminal, is designated as L.

The terminal capacitances C_L and C_H can be measured directly with the bridge, of which the decade box is a part. Consider the ratio arm B of Figure 2. It is shown with the L terminal and the shield connected to the junction of the ratio arms. Hence C_L is short-circuited, and C_H parallels the resistance R_B and does not affect the capacitance balance. If the bridge is balanced with

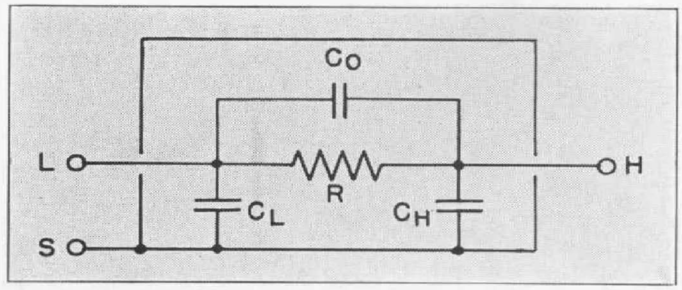


FIGURE 3. Simple equivalent circuit of a TYPE 602 Decade-Resistance Box with its associated capacitances. The panel terminal located nearest to the shield terminal is designated as L.

the P arm open circuited a value for C_{Po} will be obtained, as previously. If the shield is connected to ground, C_H is thrown across the P arm, in parallel with C_{Po}, and the bridge at balance will indicate the value of $C_H + C_{Po}$. The difference of the two readings is clearly C_H. By interchanging the L and H terminals of the box and following the same procedure, the value of C_L is obtained.

A series of measurements on a TYPE 602-L Decade-Resistance Box (a four-dial box with a maximum resistance of 111,100 ohms) show that, while C_L and C_H both vary with the resistance setting of the decade, their sum is constant. Comparison of these data with previous studies[2,3] of the TYPE 602 indicates that the capacitance C_o is of the order of 5 $\mu\mu$f, and at audio frequencies is practically constant, so that the major portion of the capacitance shunting R_B is C_L or C_H, depending upon which terminal of R is tied to the shield.

Let us examine the standard arm N of Figure 2. With the connections as shown the shields of both the decade box and the standard condenser are grounded, the capacitance C_L of the decade is thrown across the standard, and C_H parallels the series combination of R_N and $(C_N + C_L)$. It can be shown, however, that if C_N is large compared to C_H +

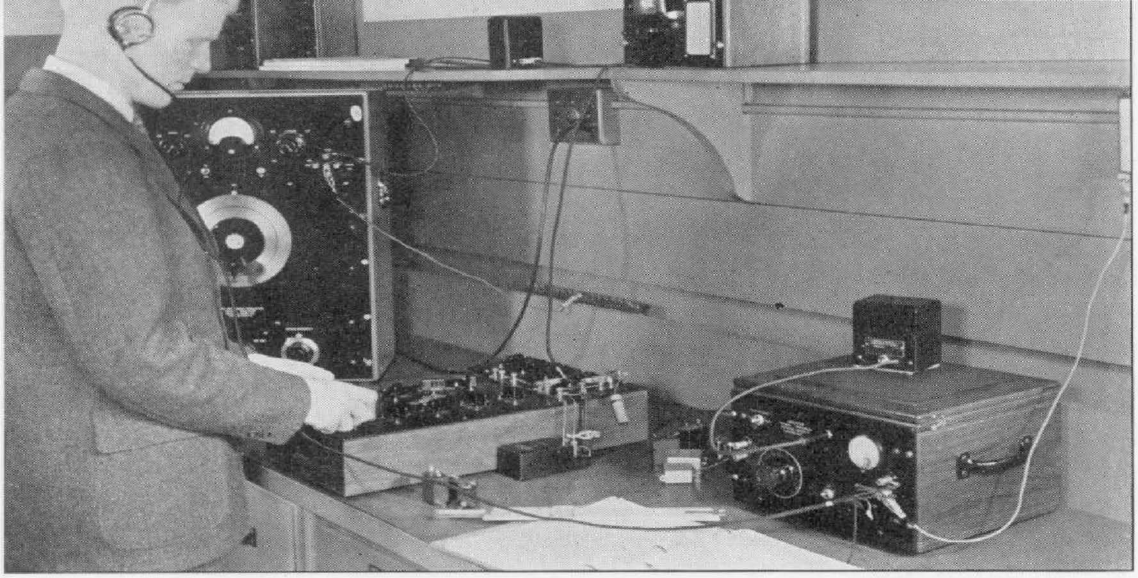


FIGURE 4. View of the series-resistance capacitance bridge assembled on a laboratory bench. The TYPE 578-A Transformer is on the shelf above the bridge.

C_L, and if the resistance of the arm is not large compared to the reactance of C_N, then the arm can be considered as a capacitance equal to $C_N + C_H + C_L$ in series with R_N. Now, knowing C_{sg} from the measurements previously outlined, and measuring $C_L + C_H$ as described above (or assuming a nominal value of 15 $\mu\mu$f per decade) we have the capacitance of the standard arm to an accuracy that will be limited solely by the accuracy to which we know the value of the standard condenser itself.

There remains the effect of the terminal-to-shield capacitances placed across the ratio arms. With the shields of the ratio-arm decade boxes connected to the junction of A and B the capacitance C_H of each arm is thrown in parallel with that arm, and the capacitance of the shield to ground is placed across the detector terminals, where it is harmless. As may be seen from an inspection of the complete balance equations of Figure 1, capacitance in parallel with a ratio arm has only a second-order effect on the capacitance balance, provided that the resultant Q's of the ratio arms are small. At a frequency of 1000 cycles the Q of a decade box set at 100,000 ohms is less than 0.01 and can safely be ignored so far as the capacitance balance is concerned.

DISSIPATION-FACTOR BALANCE

The capacitances across the ratio arms have an important effect on the dissipation-factor balance. To a first approximation the equation governing this component of the balance may be written as

(3) $$D_P = D_N + Q_A - Q_B.$$

It is, of course, desired to compute D_P from the equation $D_P = D_N = R_N \omega C_N$. To be able to do this with reasonable accuracy it is necessary to keep the difference $Q_A - Q_B$ small with respect to D_P.

At this point it may be well to discuss the effect of small residual inductances in the ratio arms. Figure 5 is a representation of a resistance with inductance in series, shunted by a capacitance. The Q of this circuit can be expressed as

(4) $$Q = \frac{\omega L}{R} - R\omega C$$

provided that $\omega^2 LC \ll 1$. Hence the series inductance introduces a Q term opposite in sign to that introduced by the shunt capacitance,* and the equations previously written may still be used by substituting for Q_A and Q_B their values modified by the presence of the series

*Grover has suggested a bridge utilizing series inductance in the ratio arms to balance for dissipation factor. Since the inductive Q is negative in terms of the equations we have written, resistance in the P arm is balanced by series inductance in the B arm.

inductance. For high resistance settings of a decade box[2,3] the inductive Q is generally completely negligible compared with the Q contributed by the shunt capacitance. At low settings, however, the inductance will predominate, and the error in the dissipation factor due to that particular arm will change its sign.

From Equation (3) it is clear that the accuracy that can be obtained by computing the dissipation factor directly from the value indicated by the setting of R_N will be limited by the sum of the effective residual Q's and D's of the network, so that $Q_A - Q_B + D_{No}$ (where D_{No} is the residual dissipation factor of the standard arm), will be the error encountered. We must allow for the possibility that all three terms are additive in the worst case. What then is the order of magnitude of the error that may be expected?

A good mica standard in the N arm will of itself have a power factor less than 0.0005, but this may be somewhat increased by the losses in C_{sg} shunting it. At one kilocycle the Q of a TYPE 602-J Decade Box (for instance) will vary roughly from -0.0008 at 10,000 ohms to $+0.0007$ at 10 ohms, so that the difference $Q_A - Q_B$ may range from zero to 0.0015. The maximum error to be expected, then, in measuring the total dissipation factor of the P arm, will be of the order of 0.0025. If the ratio arms are nearly equal, however, the difference between their Q's will be small, and the maximum value of $Q_A - Q_B + D_{No}$ may be as low as 0.001. If $Q_A - Q_B$ be made equal to D_{No} the error will approach zero. This condition may be approached by connecting a capacitor of known dissipation factor in the P arm, setting the resistance box in the N arm to the proper value, and balancing the bridge by means of a variable condenser across the A or B arm, as may be required. A known resistance in series with a known capacitance, or an air condenser of extremely small dissipation factor, may be used in the P arm.

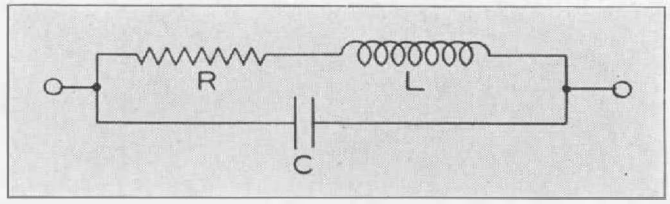


FIGURE 5. Simple representation of a resistor having residual inductance and capacitance.

This latter procedure must be followed through for each setting of R_B that is to be used and is a relatively long and tedious procedure that is generally not justified. If it is followed with care, however, the only remaining error is that caused by the variation of the Q of the A arm (something of the order of ± 0.0005 at 1000 cycles).

In all cases, if best accuracy is desired, two balances must be made, one with the unknown disconnected. Allowance for the initial capacitance and dissipation factor of the bridge can easily be made, as follows:

$$C_X = C_2 - C_1$$

$$D_X = \frac{D_2C_2 - D_1C_1}{C_X}$$

If accurate measurements of small power factors are desired, however, it is better to use a substitution method with a good calibrated air condenser in the N arm, either in the circuit described above or in the Schering bridge arrangement to be discussed in a later issue.

— IVAN G. EASTON

REFERENCES

[1]Radio Engineering Handbook, 3rd Edition, page 179, section 7.

[2]D. B. Sinclair, "Radio-Frequency Characteristics of Decade Resistors"— General Radio *Experimenter*, Vol. XV, No. 6, December, 1940.

[3]R. F. Field, "Frequency Characteristics"— General Radio *Experimenter*, Vol. VI, No. 10, February, 1932.

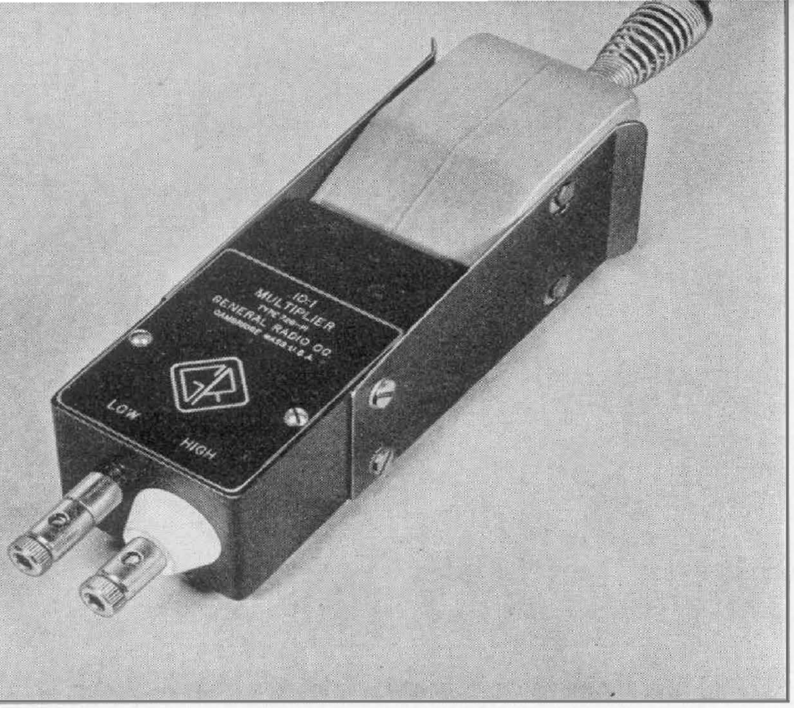


FIGURE 1 (*above*). View of the TYPE 726-P1 Multiplier with the voltmeter probe plugged in.

A MULTIPLIER FOR THE VACUUM-TUBE VOLTMETER

● **THE TYPE 726-P1 MULTIPLIER** was developed after the publication of our current catalog and, consequently, many recent purchasers of the TYPE 726-A Vacuum-Tube Voltmeter may not be aware that a multiplier is available.

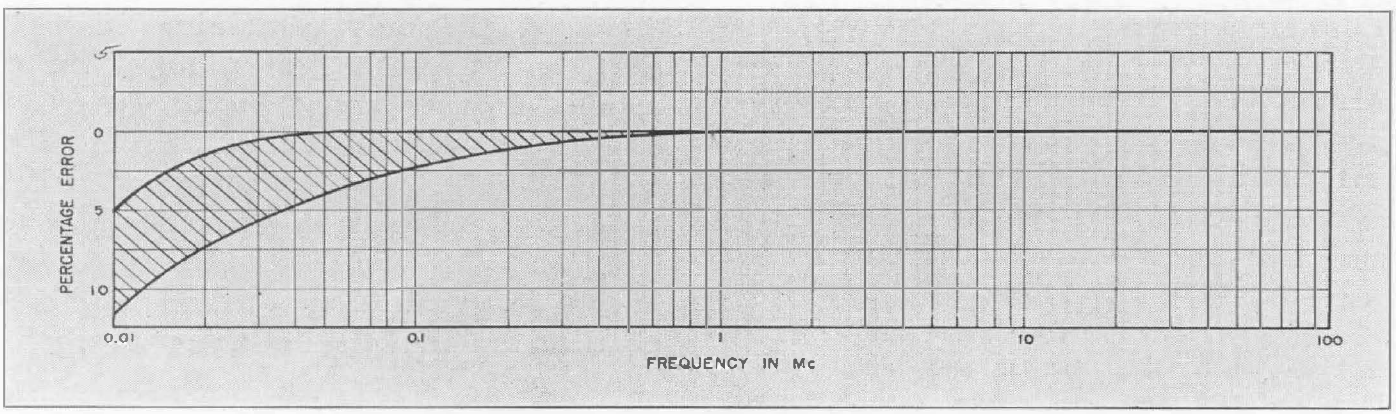
This accessory to the vacuum-tube voltmeter, which was described in the May, 1940, issue of the *Experimenter*, extends the upper limit of voltage measurement to 1500 volts.

The multiplier is a capacitive voltage divider with a ratio of 10 : 1, and is intended primarily for use at frequencies above one megacycle. The frequency error is shown in Figure 1; the mechanical features in Figure 2.

Type		*Code Word*	*Price*
726-P1	Multiplier	ALOUD	**$15.00**

Delivery can be made from stock.

FIGURE 2 (*below*). Plot of the error in multiplier ratio as a function of frequency. The input admittance of the multiplier is equivalent (between 100 kc and 100 Mc) to that of a 4.5 $\mu\mu f$ condenser of less than 0.5% power factor. The shaded area shows the variation in low-frequency error with different voltmeters.



SERVICE AND MAINTENANCE NOTES

● **WE HAVE RECEIVED** so many requests for service and maintenance notes that it is impossible for us to acknowledge them individually. The notes are now in preparation and will be mailed early in the fall.

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